Thermodynamic Optimization of Tubular Space Radiators

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Nomenclature

Bo = Boltzmann number, nondimensional, defined in the text
C_P = specific heat capacity of the fluid, J/kg K
d = diameter of the tubular radiator, m
f = friction factor, nondimensional
h = heat transfer coefficient, W/m^2 K
k = thermal conductivity of the fluid, W/mK
L = length of the tubular radiator, m
m = mass flow rate of the fluid, kg/s
N_{RC} = radiation convection parameter, nondimensional, $\sigma(T_{in}^4 - T_O^4)PL/[mC_P(T_{in} - T_O)]$
P = perimeter of the tubular radiator, πd, m
Q = heat transfer rate, W
Re = Reynolds number based on diameter, nondimensional
S = entropy generation rate, W/K
T = temperature, K
T_R = temperature ratio, nondimensional, T_O/T_{in}
V = velocity of fluid, m/s
X = horizontal coordinate, m
Δp = pressure drop, N/m^2
ε = total hemispherical emissivity of the radiator surface
ρ = density of fluid, kg/m^3
σ = Stefan–Boltzmann constant, 5.67×10^{-8} W/m^2 K^4
ϕ = nondimensional heat transfer rate, $Q/\pi kL(T_{in} - T_O)$

Subscripts
f = pertaining to the fluid
in = pertaining to the inlet
O = pertaining to outer space
opt = pertaining to the thermodynamic optimum
S = pertaining to the surface of the tube
X = at a location X along the tubular radiator

I. Introduction

PHENOMENAL advances have taken place in several areas of heat transfer in the recent past. The development of newer and faster computing machines, and availability of better experimental facilities have transformed heat transfer into a mature engineering science, with applications in such diverse areas as energy conversion, aerospace engineering, biomedical engineering, etc. However, a disturbing trend in these developments is the slow but steady delinking of heat transfer from an equally important science, thermodynamics. With energy becoming dearer and dearer, it is essential that research studies in heat transfer also address the crucial aspect of the second law analysis of the processes under consideration. This so-called synergy between heat transfer, thermodynamics and, fluid mechanics has been discussed previously.[1-3]

The problem of thermodynamic optimization of a circular tube with turbulent flow inside it and heat transfer by convection to the outside of the tube has been considered by Bejan.[4] However, the problem of thermodynamic optimization of a similar geometry, but with heat transfer to the outside by surface radiation and not forced convection, as, for example, in space radiators, has up to now not been considered. A numerical investigation of entropy generated in such a geometry, caused by both fluid friction and heat transfer, is carried out in the present study.

One of the major objectives of this study is to ascertain whether an optimum diameter exists for the radiator. The optimum diameter, from the viewpoint of an entropy generation analysis, would be a diameter that results in the least total entropy generated, when all other parameters are held fixed. In the present study, a detailed numerical investigation is carried out by varying the pertinent parameters over a certain range, and an optimum is searched for each set of the parameters. The optimum diameter does not represent the diameter at which the heat transfer rate is maximum. Two correlations are presented, one for calculating the optimum diameter and another for predicting the overall heat transfer rate from the system. To the best of the authors' knowledge, this work is the only one of its kind, for the problem under consideration.

II. Problem Definition and Solution Procedure

The problem considers the flow of a fluid through a circular pipe. The inlet temperature and mass flow rate of the fluid are specified and the tube has a surface emissivity ε. The outside ambient is characterized by specifying a sink at 3 K (outer space). The details of the problem geometry can be seen in Fig. 1. The flow inside the tube is assumed to be fully developed and turbulent, with a specified f and a specified h. Under these conditions, an energy balance on a slice of the circular tube yields the following (see Fig. 1):

$$-mC_P \frac{dT_f}{dX} = \varepsilon\sigma P(T_S^4 - T_O^4) \tag{1}$$

Here, T_S will, in general, be different from $T_{f,X}$, unless the heat transfer coefficient becomes infinitely large. However, in such cases where h is finite, an additional equation is required to evaluate T_S. This is given by

$$(T_f - T_S) = (\varepsilon\sigma/h)(T_S^4 - T_O^4) \tag{2}$$

The coefficient h, appearing in Eq. (2), was evaluated by the well-known Dittus–Boelter correlation.[5] A simultaneous solution of Eqs. (1) and (2) yields both T_f and T_S as functions of X. With this information, one is also in a position to compute the total heat transfer rate from the radiator, given the length, emissivity, mass flow rate, T_{in}, and the appropriate thermophysical properties of the fluid.

An analytical solution to Eqs. (1) and (2) is possible and straightforward.[6] Nevertheless, it is avoided here, because such a solution results in a complicated expression for T_f, which is unwieldy and, more importantly, the resulting solution is implicit and the fluid temperatures have to be determined from a solution of transcendental equations that result from the expression for T_f. It is clear from the previous discussion that even an exact mathematical solution will be no more accurate than the accuracy of the numerical procedure involved in determining the roots of these transcendental equations.

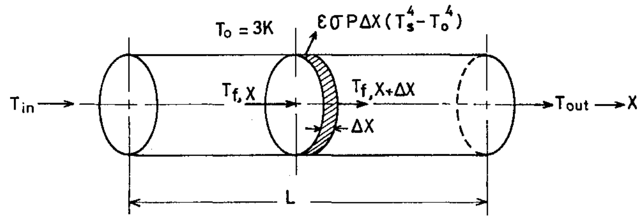

Fig. 1 Problem geometry showing energy balance for a tube element.

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In consideration of these, an explicit finite difference method was used in the present study to obtain solutions to Eqs. (1) and (2). A very fine grid (100 grid points along the length of the radiator) was used for the previous purpose. However, for validation, a representative case (not presented here) was taken to compare the analytical and numerical solutions. The agreement between the two was found to be very good.

Once the temperature distribution along the radiator is determined, the local entropy generation rate resulting from heat transfer can be calculated using the expression

$$S_{\text{heat}} = \int_0^L mC_P \frac{\mathrm{d}T_f}{\mathrm{d}X}\left(\frac{1}{\bar{T}_f} - \frac{1}{T_O}\right)\mathrm{d}X \tag{3}$$

where $\bar{T}_f$ is the average fluid temperature in the element under consideration. The local Q can be calculated from the relation $Q = mC_P\Delta T_f$. The entropy generation rate caused by the fluid flow is calculated using the following relation:

$$S_{\text{fluid flow}} = m\Delta p/\rho T_O \tag{4}$$

where Δp is given by $\Delta p = (4fL/d)(\rho V^2/2)$, and f is given by[5] $f = 0.046/Re^{0.2}$. The sum of the entropy generation rates resulting from heat transfer and fluid flow, is the total entropy generated in the system. This will be a function of the mass flow rate, emissivity, and the temperature at the inlet, apart from the properties of the fluid.

III. Results and Discussion

For a coolant fluid like air, with $T_{in} = 350$ K and $T_0 = 3$ K, calculations showed that there is a particular diameter d_{opt} for each m, ε pair for which the total entropy generated in the system is a minimum. Such a diameter will be the most desirable, because it represents the situation in which the total irreversibility in the system is the least. For a particular m, ε pair, it was observed that this optimum is very weakly dependent on L. The analysis was then extended to study the effect of fluid inlet temperature on the optimum diameter. It was observed that with an increase of the fluid inlet temperature, the optimum diameter decreases. The results are conveniently represented as a correlation in nondimensionalized form. An appropriate dimensionless number that enters the problem is the so-called Bo,[7] defined here as $Bo = \sigma T_{in}^4 L^2/mC_P(T_{in} - T_O)$. This represents the ratio of a representative blackbody radiative heat transfer rate to the maximum possible heat transfer rate in the system. Fluid flow also finds its place in this dimensionless parameter by way of m. A regression analysis then leads to the expression

$$(d/L)_{opt} = 0.0105Bo^{-0.444}\varepsilon^{-0.148} \tag{5}$$

The correlation coefficient of Eq. (5) was 0.996, the average error was ±4.5%, and the errors in the exponents of Bo and ε were ±2 and ±7%, respectively. This correlation is based on 35 data, and a parity plot shows the absence of any bias. The range of parameters for the previous correlation are $0.05 \le \varepsilon \le 0.98$ and $0.02 \le Bo \le 1.5$.

If one considers the effect of various parameters on the optimum diameter, it is seen that ε has a negative exponent of 0.148. This means that higher emissivity values shift the optimum diameter to lower (d/L) values. This is intuitively apparent, because of the fact that the higher the emissivity, with the other parameters being the same, the larger the heat transfer and, hence, the larger the entropy generation rate. Since the optimum basically arises because of the competition between entropy generated from heat transfer and that from fluid flow, the optimum is reached sooner for a higher ε (i.e., optimum is reached at a lower value of diameter). This can be seen clearly from Fig. 2.

With regard to the effect of the Boltzmann number on the optimum diameter, it is clear that a higher Bo means a lower

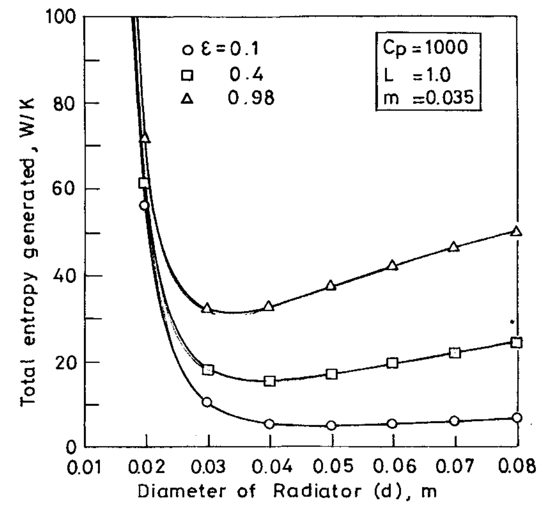

Fig. 2 Variation of total entropy generated with diameter for various emissivities.

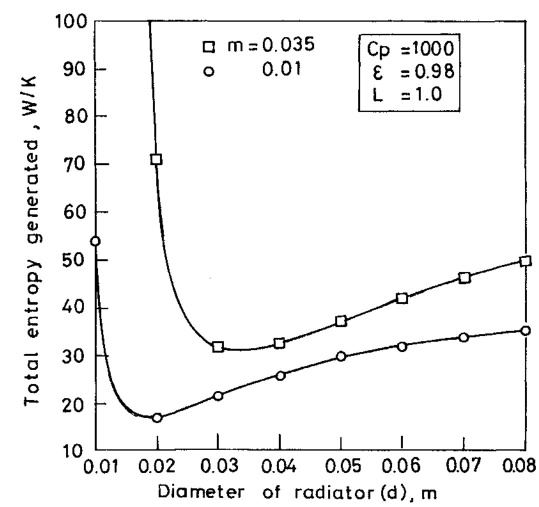

Fig. 3 Variation of total entropy generated with diameter for various mass flow rates.

m and, hence, the optimum shifts to lower values of d, because of the negative exponent of Bo. This is also evident from Fig. 3, which highlights the effect of mass flow rate on the optimum diameter. It is also apparent that a higher inlet temperature will shift the optimum to lower values, in view of the increased entropy generation rate caused from heat transfer, with the entropy generated from fluid friction remaining the same. Equation (5) confirms this.

The effect of L on the optimum is interesting. Bo contains the term L^2 and the correlation [Eq. (5)] shows that $(d/L)_{opt} \propto (Bo)^{-0.444}$. This means $(d/L)_{opt} \propto L^{-0.888}$ or $d_{opt} \propto L^{0.012}$; therefore, d is a weak function of L. In the range of lengths considered in the present study ($0.5 \le L \le 2$), the maximum effect of L on d_{opt} is around 1%.

Based on some 80 data, a correlation was evolved for computing Q with d_{opt} obtained from Eq. (5) for the given length, m, C_P, and ε for a particular fluid. This is given by

$$\phi = 0.000125Re^{1.00}N_{RC}^{0.183}\varepsilon^{0.691} \tag{6}$$

The ranges of parameters for the correlation are $0.01 \le d \le 0.09$, $0.5 \le L \le 2$, and $3 \times 10^4 \le Re \le 10^6$. The previous equation has a correlation coefficient of 0.998 and has an error band of ±2%. The exponent 0.691 for ε shows a strong dependence of ϕ on ε, and the N_{RC} term brings out the convective–radiative interaction. The exponent of Reynolds number is unity and this exponent would have been 0.8, for turbulent-forced convection flows. The dimensionless parameter ϕ, from the way it is defined here, actually represents some sort of a Nusselt number for the given situation and, hence, it is not surprising that it can take values greater than unity. A parity plot showed that the correlation represents the data well and does not have any bias.

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Internal Radiation Effects in Zirconia Thermal Barrier Coatings

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Nomenclature

a = absorption coefficient of semitransparent coating, m^{-1}
c_0 = speed of electromagnetic propagation in vacuum, m/s
G = flux quantity $2(q_r^+ + q_r^-)$, W/m^2; $G_L = G/\sigma T_{g1}^4$ in large-frequency spectral band
h_1, h_2 = convective heat transfer coefficients at boundaries, W/m^2K; $H = h/\sigma T_{g1}^3$
K = extinction coefficient of semitransparent material, $a + \sigma_s$, m^{-1}
k_c, k_m = thermal conductivity (coating, metal), W/mK; $N_c = k_c/\sigma T_{g1}^3 \delta_c$
n = refractive index of semitransparent material
q = heat flux, W/m^2; q_r = radiative flux, W/m^2; $\tilde{q} = q/\sigma T_{g1}^4$
q_r^+, q_r^- = isotropic radiative fluxes in $+x$ and $-x$ directions, Fig. 1, W/m^2
T = absolute temperature, K; $t = T/T_{g1}$
T_{g1}, T_{g2} = gas temperatures on hot and cold sides of composite, Fig. 1, K
T_{s1}, T_{s2} = temperatures of surroundings on two sides of composite, Fig. 1, K
x = coordinate in composite, m; $X = x/\delta_c$
δ_c, δ_m = thicknesses of coating and metal, m
ε = emissivity
κ_{cL} = optical thickness of coating in large-frequency band, $K_L\delta_c$
λ, ν = wavelength and frequency of radiation; ν_{co} = cutoff frequency
ρ = diffuse reflectivity of interface from Fresnel equations
σ = Stefan–Boltzmann constant, W/m^2K^4
σ_s = scattering coefficient in semitransparent coating, m^{-1}; $\Omega = \sigma_s/K$

Subscripts

c = coating
i = inside interface at the hot side of coating, Fig. 1
L = in spectral band where $\nu > \nu_{co}$
m = metal
mc = cold side of the metal wall
mh = hot side of the metal wall
o = outside interface at the hot side of coating, Fig. 1
so = soot
ν = frequency dependent

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Introduction

USING thermal barrier coatings on combustor liners, turbine vanes, and rotating blades is important for reducing metal temperatures in current and advanced aircraft engines. Zirconia is a common coating material, and it is partially transparent to thermal radiation. Radiation becomes more significant as temperatures are raised for higher efficiency in advanced engines. Calculations are often made with radiation effects neglected inside the coating. The effect of radiation is illustrated here, where an analytical procedure is provided by using the two-flux method for the radiative contribution.

In Ref. 1 a detailed study was made of ceramic thermal barrier coatings for diesel engines. In Ref. 2 a two-flux analysis was developed for radiation in semitransparent multilayer composites. These references provide the basis for the present analysis where illustrative solutions are obtained for typical conditions in an aircraft engine. The formulation and solution of the exact spectral radiative transfer equations including large scattering, as is characteristic of zirconia, are rather complicated. For plane-layer composites, the two-flux equations were shown in Ref. 2 to give accurate results for gray and spectral layers; hence, the two-flux method is used here to provide a simplified method.

Analysis

A composite is first considered of a thermal barrier coating on a metal wall with the external surface of the coating covered with a thin opaque layer of soot (upper part of Fig. 1). For an advanced engine, the combustion chamber pressure is

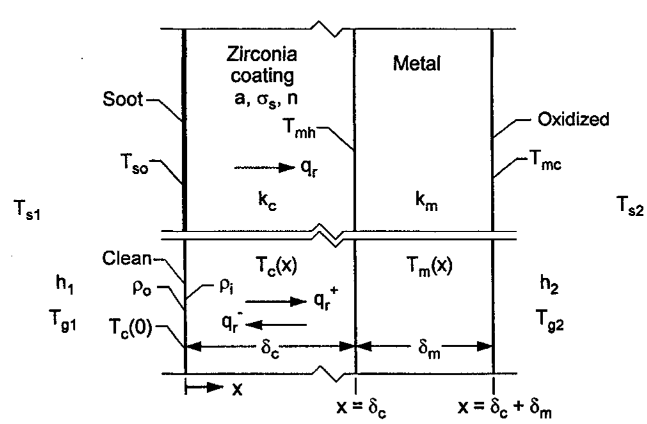

Fig. 1 Geometry and nomenclature for a thermal barrier coating on a metal wall with and without soot on the exposed surface of the coating.